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Notes on the Quintic.

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1. If the quintic

$$x^5 + 10p_2x^3 + 10p_3x^2 + 5p_4x + p_5 = 0$$

be solvable by radicals, the coefficients must be so related that if

$$p_2 = n\theta k^2, \qquad p_3 = \alpha\theta^2 k^3, \text{ and } p_4 = \beta\theta^2 k^4,$$

then must

$$p_5 = 2(1+n)\gamma\theta^2 k^5,$$

wherein

$$\theta = (1+n)\{1-(1+m^2)n^2\}$$

$$\alpha = 4g\{m-(1+n)g\}/[\{1+m^2-(1+n)g^2\}^2-(1+m^2)]$$

$$\beta = \frac{3-4m}{1+m^2} + 4n + 5(1+m^2)n^2 + 4(1+n)\{m-(1+m^2)n-(1-mn)g\}\alpha,$$

$$\begin{aligned}
\gamma = 2(1+n)\,[&1+m+2m^2+m^3-2(1+m^2)(1+m+m^2)n+m(1+m^2)n^2\\
&-\{1+m+m^2+(1-2m-2m^3)n+(1+m^2)^2n^2\}g\\
&-(1+n)\{m-2(1+m^2)n+m(1+m^2)n^2\}g^2\\
&+(1+n)\{1-2mn+(1+m^2)n^2\}g^3]\,\alpha^2\\
+\Big[&\frac{2}{1+m^2}-3(5+2m)+2(5+6m)n+(1+m^2)(19+6m)n^2-(1+m^2)(3+4m)n^3\\
&+4(1+n)\{1-2mn+(1+m^2)n^2\}g\\
&+(1+n)\left\{\left(\frac{1}{1+m^2}-n^2\right)\left(\frac{11+2m}{1+m^2}-4n\right)+4\left(\frac{m}{1+m^2}-n\right)n^2\right\}g^2\Big]\,\alpha\\
+2\Big[&\{1-(1+m^2)n^2\}\left(\frac{11+2m}{1+m^2}-4n\right)+4\{m-(1+m^2)n\}n^2\Big].
\end{aligned}$$

2. If $g = 0$, the solvable quintic becomes

$$\begin{aligned}
x^5 + 10n\theta k^2x^3 + 5\left\{\frac{3-4m}{1+m^2}+4n+5(1+m^2)n^2\right\}\theta^2k^4x&\\
+4\left[\left(\frac{11+2m}{1+m^2}-4n\right)\theta+4\{m-(1+m^2)n\}(1+n)n^2\right]\theta^2k^5 &= 0.
\end{aligned}$$

3. If both $g = 0$ and $n = 0$, the solvable quintic assumes the form

$$x^5 + 5\left(\frac{3-4m}{1+m^2}\right)k^4x + 4\left(\frac{11+2m}{1+m^2}\right)k^5 = 0,$$

a form communicated to the present writer by Professor G. P. Young, of Toronto University, in May, 1883. As any quintic can by means of the Jerrard-Tschirnhausen transformation be reduced to the form $x^5+ax+b=0$, Professor Young's form is equivalent algebraically to the form given in §1 above.

4. If in §1 $m=\frac{p^2-1}{2p}$, the resulting quintic includes Cockle's solvable quintic (Lady's and Gentleman's Diary, 1858) which itself includes the Binomial, the DeMoivrian (1706) and the Euler-Bezout (1762) quintics. For example, if we make

$$g=0,\ m=\frac{p^2-1}{2p} \text{ and } n=-\frac{2p(2p+1)}{p^2+1}$$

we get DeMoivre's quintic.

In the *Quarterly Journal of Mathematics*, Vol. XVIII, pp. 154–157, Professor Cayley, apparently unaware that he had been anticipated by Mr. Cockle, reduces the solution of Cockle's quintic to depend on that of a cubic, but in reality the solution can be obtained without solving any equation of higher degree than a quadratic.

5. The Gaussian quintics for the solution of the binomial

$$x^n-1=0,\ n=10m+1,$$

m being prime to 5 and n a prime number, can be written

$$x^5-10nx^3-5dnx^2-5enx-gn=0,$$

wherein

$$\begin{aligned} d&=a^2+ab-b^2\\ 16n&=d^2+25(a^2+b^2+5)\\ 16e&=3d^2-25(a^2+b^2+25)\\ 16g&=\{d^2-25(a^2+b^2+5)\}d-1250ab.\end{aligned}$$

If m be a multiple of 5,

and $$4d^2=(a^2+b^2)^2,$$

then $$g=(e-n-125)d.$$

The following table exhibits the values of a, b, d, e and g for all prime values of n under 200:

n	a	b	d	e	g
11	1	1	1	—42	—89
31	3	1	11	—32	—409
41	3	—3	—9	—52	981
61	5	—3	1	—92	1111
71	1	5	—19	—12	101
(101)	(1)	(—5)	(—29)	(78)	(271)
131	5	7	11	—132	—4009
181	9	—5	11	—182	7059
191	7	—1	41	198	1331
101			21	—22	—2604
151	7	3	—29	28	1798